Self-organized chaos through heterostatic optimization

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Entropy and heterostasis

Heterostasis

- Self-regulating processes aimed at stabilizing a certain target distribution of dynamical behaviors.
- Contrast to homeostatic regulation which aims at stabilizing a steady-state dynamical state.

Maximization

- Maximization of the entropy of neuron firing rate distribution has important implications
  - Uniform usage of all the output activity states.
  - Increase of the information transfer between the input and the output states.
- Entropy maximization of neural output activity is limited by the energy resources available to a neuron.

Constrain on the output distribution

- Fixed average energy consumption

$$\int_0^1 p(y) f_E(y) dy = \mu \,, \qquad (1)$$

where $f_E(y)$ is an energy usage of a neuron as a function of the output firing rate y

Target distribution

- Entropy maximizer on the finite interval [0,1], and with the constrain mentioned above, has the following form

$$p_{exp}(y) = \frac{1}{h(\lambda)} e^{-\lambda f_E(y)}$$

where $h(\lambda) = \int_0^1 e^{-\lambda f_E(y)} dy$, and $\mu = \frac{\partial}{\partial \lambda} \ln(h(\lambda))$.

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Learning rules

Neuron transfer function

- Sigmoidal transfer function

$$y(t+1) = g(x(t)) = \frac{1}{1 + e^{-(a(t)*x(t)+b(t))}} \,,$$

where $x(t)$ is input rate at previous time step.

Learning rules

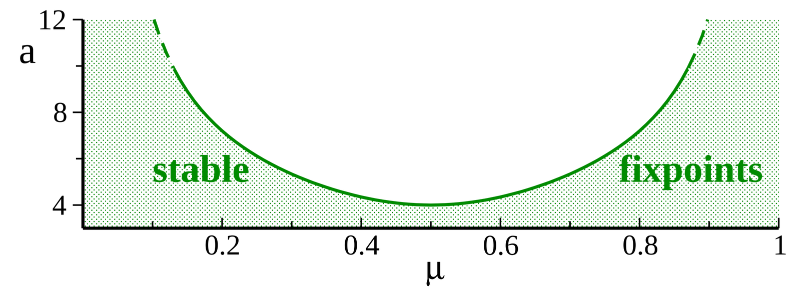
- Learning rules for the intrinsic plasticity are obtained by using the gradient descent on the Kullback-Leibler divergence, between the target distribution $p_{exp}(y)$ and the output probability distribution $p_y(y)$, with respect to internal parameters a and b

$$\begin{aligned} b(t+1) &= b(t) + \epsilon_b \Delta\tilde{b}(t) \\ a(t+1) &= a(t) + \epsilon_a \left(1/a(t) + x(t)\Delta\tilde{b}(t)\right) \\ \Delta\tilde{b}(t) &= 1 - (2 + \lambda f_E'(y(t+1)))y(t+1) + \\ &\quad + \lambda f_E'(y(t+1))y^2(t+1) \end{aligned}$$

Stability analysis

- For the slow changes of $a(t)$, therefor $\epsilon_a \to 0$, a solution of the $\Delta\tilde{b}(y^*) = 0$ is a stable fix point of the system if the following relation is satisfied

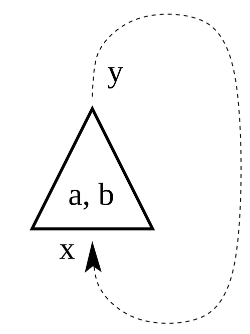
$$a < \frac{1}{y^*(1-y^*)} \,.$$



Simulation

- We have chosen linear dependency of energy depletion with respect to neuron firing rate

$$f_E(y) = y \,.$$



- In the case of the single site loop we use the balanced substitution

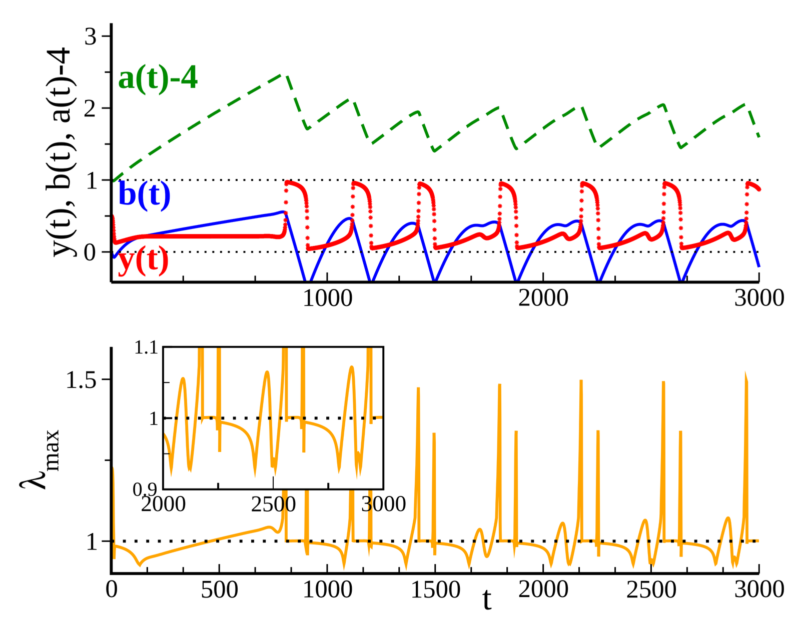
$$x(t) = y(t) - \frac{1}{2}$$

Simulation - single neuron

Setup

- Target mean output activity $\mu = 0.28$, thus $\lambda = 3.017$.
- slow learning rates $\epsilon_a = \epsilon_b = 0.01$

System dynamics



- Stability analysis along the trajectory shows that maximal local Lyapunov exponent oscillates between frozen and chaotic phase.

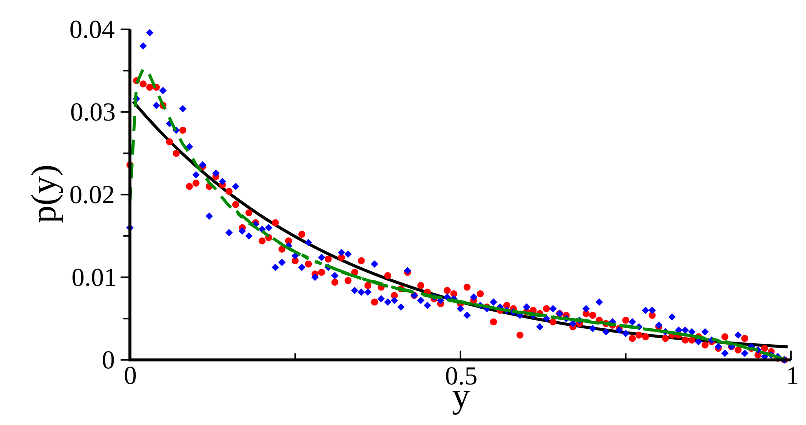
Simulation - neural network

Setup

- Fully connected network with $N = 500$ neurons
- Synaptic weights are drawn from

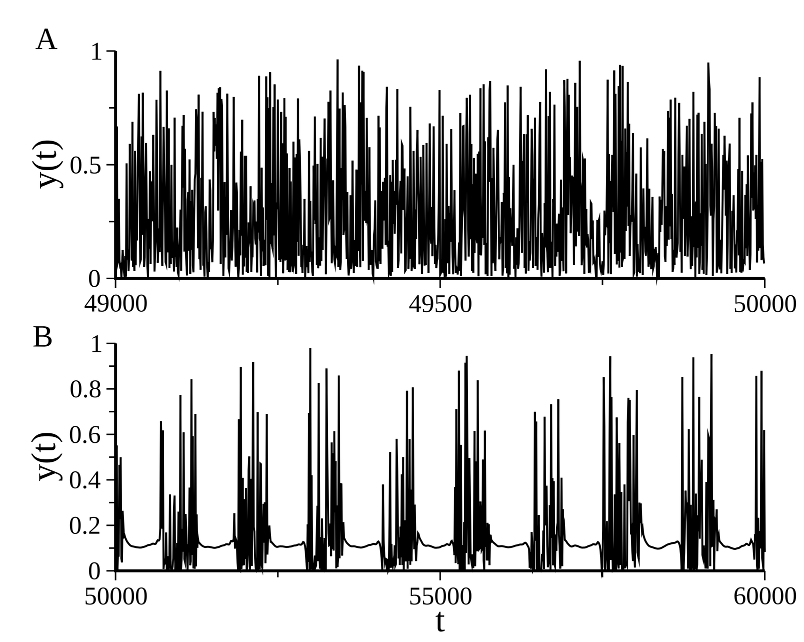
$$p(\omega) = \frac{1}{2}\Big(\delta(w - \frac{1}{\sqrt{N}}) + \delta(w + \frac{1}{\sqrt{N}})\Big)$$

Firing rate PDFs



- Output distributions of the two neurons with highest (blue diamonds) and lowest (red circles) Kullback-Leibler divergence ($D = 0.03$ and $D = 0.15$) compared to the mean output distribution (dashed green line) and the target exponential output distribution (full black line).
- Target mean firing rate $\mu = 0.28$, and $\epsilon_a = \epsilon_b = 0.01$

Output activity



- Output activity of a randomly chosen neuron in a fully connected network.
- The target average firing rate, for all the neurons in the network, is $\mu = 0.28$ (A) and $\mu = 0.15$ (B).

References

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